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DIMINUTION OF MALEVOLENT GEOMETRIC GROWTH THROUGH INCREASED VARIANCE

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ABSTRACT

Often, geometric growth maximization techniques as presented by (Kelly, 1956; Thorp, 1962; Vince et. al., 2019 & Vince, 1990) have looked only at such ambits where the implementor was concerned with maximizing growth, or more generally, situations where growth was beneficial to the implementor (Vince, 2013; Vince, 2015).

In this paper we examine using these techniques on geometric growth functions where the implementor benefits from diminished growth. Certain geometric growth functions accruing against the public, often characterized as "growing out of control," typically meet this criterion. These often include medical costs, the growth of populations (e.g., bacteria) or pathogenic infections in a population, infected cells in an organism, or even the growth of cumulative national debt.

Finally, we demonstrate the technique upon this notion of the growth rate of a nation's cumulative debt. Heretofore, debt reduction has been considered along the one-dimensional tug-of-war between reducing government services or increasing taxes. The technique presented, albeit very simple, provides a politically-agnostic means of debt-reduction. We provide further examples of potential applications in terms of mitigating malevolent cellular growth and concerns for applying the material in the spread of pathogenic outbreaks.

BACKGROUND

Interaction with stochastic, time-series data is implicitly an exercise in geometric growth maximization. If we consider a random stream of outcomes over time, in almost every case, resources available to allocate to the immediate outcome is an unavoidable result of the cumulative trail of outcomes to that point in time. The results compound. There exists not-so-common exceptions to this compounding effect (consider a retiree who receives a pension he does not need in order to meet his expenses, which he chooses to wager on a particular gamble with each pension distribution), but in most cases compounding is implicit, and we restrict our discussion to such ever-prevalent cases.

Maximization of results from stochastic, time-series data has a long history, beginning with (Bernoulli, 1738) who, in 1738, provides the first known reference to "geometric mean maximization".

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This 1738 paper, written in Latin containing the first known reference to geometric mean maximization, is translated into English in 1954 (Bernoulli, 1738).

In his 1956 paper, (Kelly, 1956) showed how Shannon's "Information Theory" (Shannon, 1948) could be applied to the problem of a gambler who has inside information in determining his growth-optimal bet size.

Six years later, (Thorp, 1962) applies the concept to the actual gambling situation presented in the game of Blackjack. Thorp would later provide closed-form solutions.

In 1990, (Vince, 1990) provides a solution for capital market applications, and by 2019 in (Vince & Optimal, 2019) provides the full (non-asymptotic, non-instantaneous) solution for geometric mean maximization for all conditions.

Other avenues where other variants of geometric growth amplification are shown to be "optimal", for criteria other than outright maximization appear in (Lopez, Vince & Zhu, 2019; Vince & Zhu, 2013; Vince & Zhu, 2015).

Yet everything up to such a point refers to geometric growth as something that is desired, beneficial, and seeks ways to maximize the benefit derived from such streams of stochastically-generated outcomes.

From (Vince & Optimal, 2019) we find, in simplest form, geometric growth is maximized for a single stream (as opposed to multiple, simultaneous streams) of Q stochastic outcomes[2], x, asymptotically (*i.e.,* as the number of trials approach infinity), by that fraction of the total cumulative pool at any i, by that value for f (0…1) which maximizes the geometric mean outcome, G:

$$G(f) = \sqrt[Q]{\prod_{i=1}^{Q} 1 + f(-x_i/\min(x_1 \dots x_Q))} \tag{1}$$

Where:

G =the probability-weighted geometric mean outcome of the stream of outcomes.
Q = the number of outcome events in the stream of outcomes.
f =the constant fraction of the total pool of resources allocated on each outcome in the stream.
x_i =an individual result of the i'th outcome event.

n.b. min ($x_1 \dots x_Q$) must be a negative number, otherwise the value for f which maximizes G(f)is 1 for all possible values of Q.

Let us examine a simple stream of outcomes which

are binomially-distributed, such as the results of a coin toss. We will assign a value of 2 if the coin lands favorably to us, and a value of -1 if it lands the opposite way, unfavorable to us.

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[1]As mentioned, the value for *f* which maximizes *G(f)* in (1) is asymptotic with respect to ever-greater *Q*. When *Q=1*, if the probability weighted mean value for x is positive, then *G* will be maximized at a value of 1, and this will approach the value given in (1) asymptotically with ever-higher *Q*. If the probability-weighted mean value for *x* is zero or less, then the function for *G* is maximized at *f=0*, and remains for all possible values of *Q*.

Thus, if one seeks to find that value for *f* which maximizes *G(f)*, equation (1) is properly expressed as:

$$\lim_{Q\to\infty} G(f) = \sqrt[Q]{\prod_{i=1}^{Q} 1 + f(-x_i/\min(x_1 \ldots x_Q))} \qquad (1)$$

The non-asymptotic formulation, that is, the determination of the growth-optimal value for *f* for a finite-length stream of stochastic outcomes ($Q < \infty$) is given in (Vince, 1995).

Figure 1. Equation (1) for *f=0* to *1*.

It is important to note that growth gets ever greater as *f* increases from 0 to its asymptotically growth-optimal value between 0 and 1, as given in equation (1) and then decreases. That is, asymptotically, as *Q* gets ever-greater, growth is reduced with ever-greater values for *f* beyond that given in equation (1).

Thus, we find for our 2: -1-coin toss, by Equation (1), we see that geometric growth of our pool is achieved as *Q*, the number of trials, gets ever-greater, at *f=.25* as we see in **Figure 1**.

If we took this stream of outcomes, *2, -1,* where *Q=2*, and applied the values to (1), we would find that *G(f)* is maximized at *f=.25* where *G (.25) =1.06066*. Thus, if we were to wager *.25* of our entire pool on each coin toss in sequence, regardless of sequence, we would maximize our gains as *Q* got ever greater.

$$G(.25) = \sqrt[Q]{(1 + .25(-2/-1))x(1 + .25(--1/-1))}$$

$$G(.25) = \sqrt[Q]{1.5\ x\ .75}$$

$$G(.25) = 1.06066$$

This value, 1.06066, the geometric mean return outcome, represents the multiple we would make on our pool, *on average*, with each coin toss, with each event, since we are "reinvesting" after each toss (*i.e.,* our pool, from which we take a fraction, *f*, of to wager on the next toss, is the cumulative result of the outcomes of all subsequent tosses). This number is necessarily less than the "average" (the arithmetic mean return per event):

$$A(f) = \frac{\sum_{i=1}^{Q} 1+f(-x_i/\min(x_1 \ldots x_Q))}{Q} \qquad (2)$$

Just as there is a geometric mean outcome, *G*, for any value for *f*, so too is there an arithmetic mean outcome, we shall designate as *A*, and for our 2:-1 coin toss, where $f=.25$, we find $G=1.06066$ and, from (2), $A = 1.125$.

SEEKING GEOMETRIC GROWTH ATTENUATION OR DIMINISHMENT

We shall now play devil's advocate, where our goal, when faced with the growth of such streams which appear with a geometric growth amplifier innate to them, is to attenuate rather than amplify such growth rates. Typically, the literature on the matter has previously been focused on maximizing growth in a stream of stochastic outcomes. Now, we shall examine ways, first introduced in (Vince, 1995), to use the ideas to reduce geometric growth in stochastic outcomes. The real-world is rife in systems where we would desire this. Among these types of phenomena are things like the growth of bacteria in a petri dish, the spread of infectious disease in a population of organisms, the spread of pathogenic cells within an individual organism, or even, say, the growth rate of a nation's cumulative debt. The same mathematical formulations are at work as when we desire growth.

The problem is, in looking for diminishment in these aforementioned types of geometric growth situations where we seek diminishment, what is the *f* value? Clearly, there is no such governing value of the growth rate of a nation's aggregate debt's rate of growth or in other situations where we desire geometric growth attenuation or diminishment. Where do we obtain an available parameter to affect the formulation which belies the geometric growth function here?

Just as there is a geometric mean outcome, *G*, for any value for *f*, so too is there an arithmetic mean outcome, we shall designate as *A*, and a variance, we shall designate as *V*, to the stochastic stream of outcomes.

For our 2: -1-coin toss, we thus have corresponding values of 1.06066 for *G*, 1.125 for *A,* and .140625 for *V*, the *population* variance. Since standard deviation, *SD*, is simply the square root of variance, we can express this $SD = 0.375$.

We can thus summarize as follows for this simple, binomial case:

$f = 0.25$

$G = 1.06066$

$A = 1.125$

$SD = 0.375$

Serendipitously, as demonstrated in (Henry, 1959), these values comport perfectly to the *Theorem of Pythagoras* as:

$$A^2 = G^2 + SD^2 \tag{3}$$

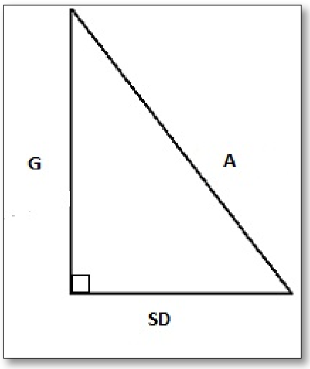

Figure 2. Pythagorean relationship of $A^2=G^2+SD^2$.

Our goal here, as the advocate of the devil, is to seek geometric growth diminishment. Thus, we seek to make the value for G be as small as possible, as it can be said that the growth of the pool after *Q*events is G^Q.

With *Q* as a given, any reduction in *SD* that can exceed an offsetting increase in *A* will reduce *G*, and, geometrically, the size of the pool after *Q* events, G^Q. From this, we can state the following:

> *With respect to growth multiples, any increase in their variance, V, affects the geometric growth rate, G, by an amount equal to an equivalent decrease in the arithmetic average - squared!*

In most human endeavors, when confronted with a malevolent geometric growth function, man's first reaction is to seek to minimize the (arithmetic) average growth rate, *A*, when equally as much beneficial fruit can be derived merely by increasing the period-on-period variance, *V*.

Variance, *V*, or it's square root standard deviation, *SD,* depending on the function, often tend to move with respect to *A* in a non-linear manner; *i.e.,* often, as *A* increases, so too does *SD*, with the latter increasing at a faster rate. This leads to the familiar peak in *G (0...1)* such as we see in **Figure 1**.

However, in many malevolent, naturally occurring growth functions, we are able to effect *SD*, to increase *SD*, without a corresponding increase in *A*. If, in **Figure 2**, one increases *SD* without a corresponding increase in *A* of the same increase as being applied to *SD*, then *G* is necessarily smaller.

Any increase in the base of a right-angled triangle without any equivalent or greater increase in the hypotenuse will reduce the vertical leg, will achieve our goal, will diminish growth.

We thus find there exists an isomorphism between the *f* value illuminated in Section 1 and the cosine of the standard deviation and mean of the period-on-period growth in a stream of malevolent stochastic events. Graphically, this is depicted in **Figure 3**.

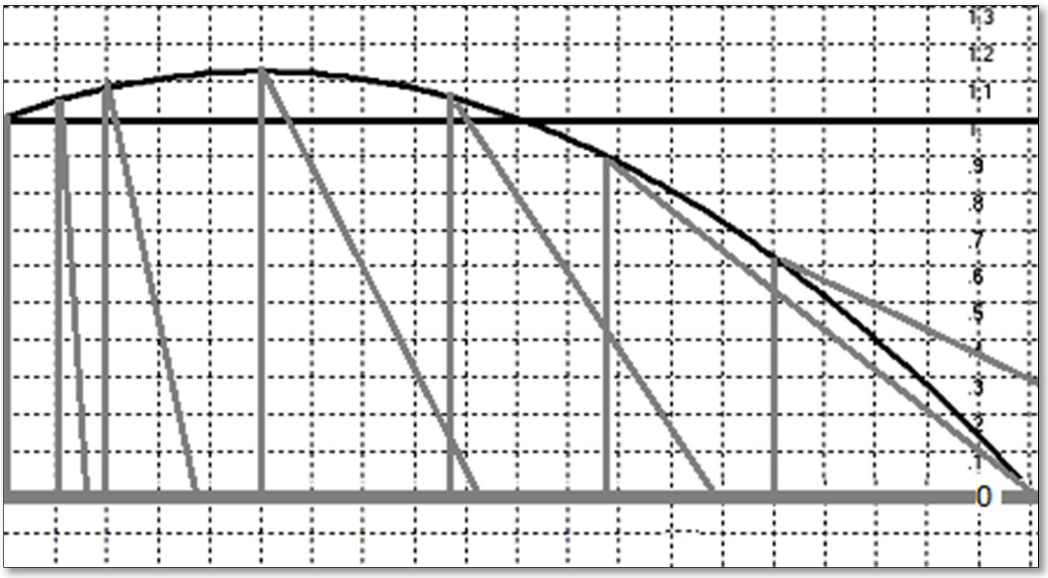

Figure 3. The isomorphic relationship to f and the cosines of SD/A.

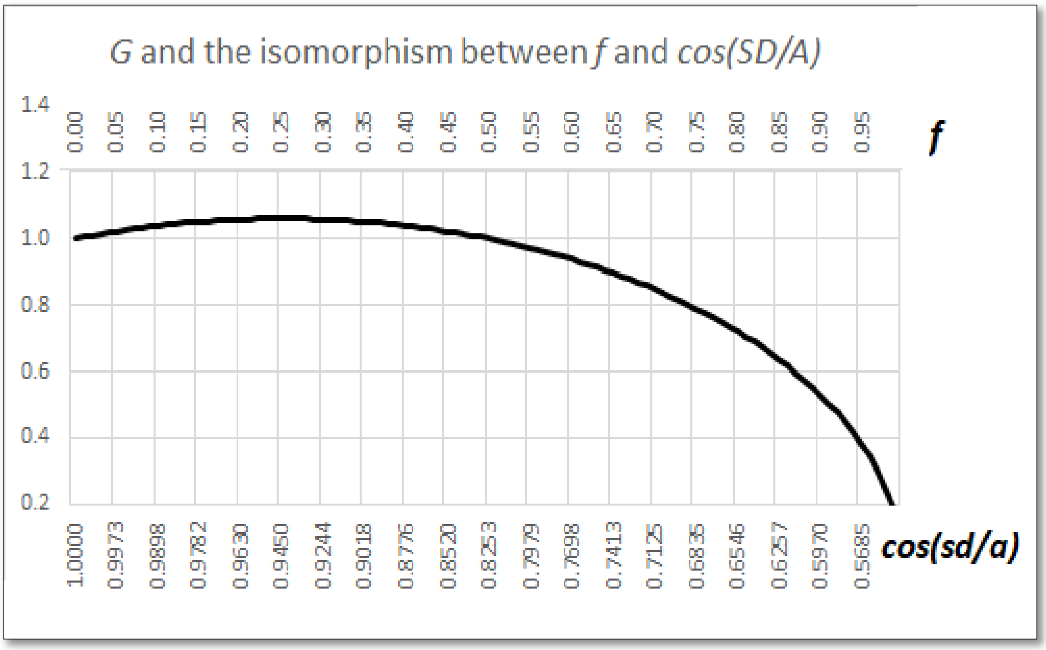

Figure 4. The isomorphic relationship to f and the cosines of SD and A.

Figure 4 bears out that the isomorphic relationship simply tells us that the cosines can be "mapped" to *f*-values, but that the numeric values aren't what matter and the values themselves along the vertical axis aren't of importance. The vertical axis simply represents the degree of some directed effort, be it by increasing the size of an allocation or increasing the variance in a stream of stochastic outcomes. It is the curve, the shape of the curve, and where we position ourselves on this curve based on our *directed effort* that matter. We are considering the fact that *the curve has a peak beyond which geometric growth begins to diminish*, and may allow us the opportunity to potentially navigate there through our directed effort.

In the simplest case, if we disregard *A* varying with respect to *V*, we can unequivocally then state that for constant *A*, that is if we can hold *A* constant, any increase in *V* reduces *G*. If *A* increases with respect to *V*, we witness a curve with a peak in the horizontal range of values beyond which geometric growth begins to diminish.

Figures 1, 3, and 4 show where *A* varies with changing *V*. This is not necessarily the case however, as often variance can be increased without increasing the arithmetic average in a sequence, particularly those of "malevolent" character where our directed efforts can be used to increase variance. When *A* does *not* increase with increasing *V*, the peak of the curve is at its left-most point, that point where $f=0$ and $cos(SD/A) = 1$ (*i.e.,* the "base" of the triangle is zero-- is it no longer a triangle at this point but merely a vertical line).

Thus, there exists two important points in the range of values of "directed effort," either with respect to percentage of a resource we allocate to the multiples of a stream of stochastic outcomes or the cosines of the standard deviation to means of these outcomes.

The two points are portrayed in **Figure 5**. Working from the left, the first of these points is that point where the curve begins descending, where the slope of the first derivative turns negative. This represents that point where as *f* (the "*fraction* allocated") increases further, or the size of the base (*V*) relative to the hypotenuse (*A*) increases further, geometric growth becomes less.

Of note, if *A* does not increase with increasing *V, the peak of the curve is at the leftmost bound*, and hence this first geometrically significant point is immediately to the right of the leftmost bound by an infinitesimally small amount; *i.e.,* as soon as *f* or *V* increases, *G* decreases.

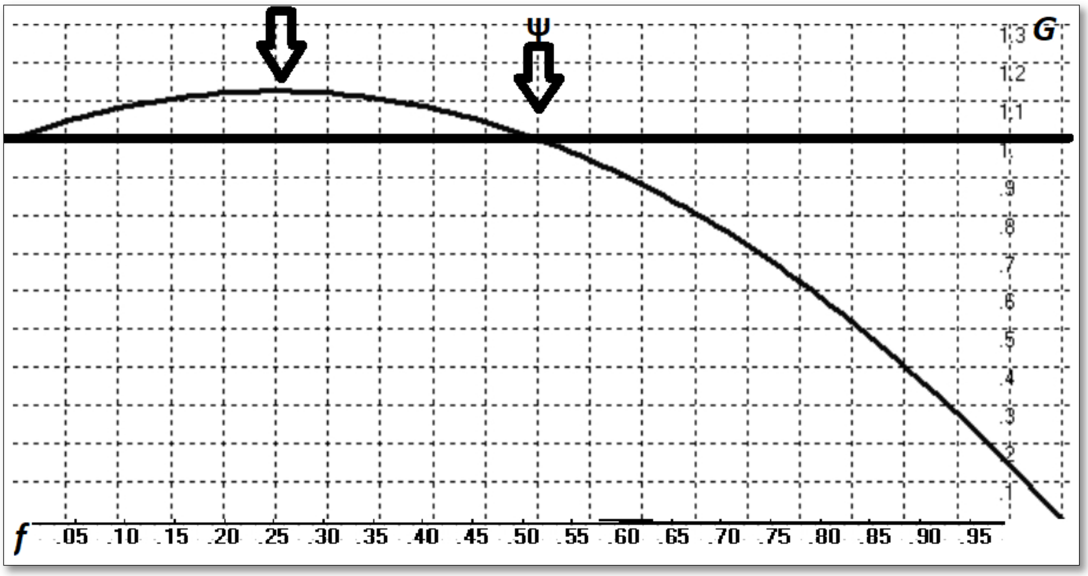

Figure 5. The two points of geometrical significance in growth diminishment. In this graphic (2: -1-coin toss) A increases with increasing V.

The second important point is that point where the value for G crosses 1 to the downside, and we refer to this point as ψ. Any given number multiplied by a number less than 1 will have a lower product than the given number. Repeatedly multiplying the resultant product by a number less than 1 will cause the resultant products to get ever-closer to zero. A value for G less than unity will cannibalize itself as more periods accrue.

From this, we can see that Sir Ronald Fisher's fundamental theorem of natural selection (Ronald, 1930), which states:

"The rate of increase in fitness of any organism at any time is equal to its genetic variance in fitness at that time."

Provides only a partial explanation; it consists of looking at the natural world as it exists in nature, to the "left" of the peak where A increases with increasing V. However, by increasing this variance beyond the peak, we can see that such fitness diminishes with further increases in variance, and at the point ψ extinction is ultimately assured.

In the natural world, the point to the right of the peak is obscured; too much variance in fitness, and the organism no longer exists.

The Pythagorean relationship in (3), expressed between the arithmetic mean of growth multiples and the geometric mean and standard deviation of these multiples, is exact if the stream of multiples is binomially distributed.

Other measures for estimating geometric growth have been proposed and can sometimes be more accurate and deserve discussion.

In (de La Grandville, 1989) speaks of misconceptions in estimating long-term returns and refers to the popularity of one widely held formula for estimating geometric returns as being "approximately equal to the arithmetic average minus half of the variance," or:

$$G = A - V / 2 \tag{4}$$

Another formula, common in quantitative finance for estimating the geometric mean return which provides an exact estimate when the returns are lognormally-distributed is:

$$G = A/\sqrt{1 + V \ x \ A^{-2}} \tag{5}$$

Generally (3) will be less than (5). Typically, (3) is not greater than the actual G for most real-world, non-binomially-distributed data sets.

All examples for estimating the geometric return, Equations (3), (4), and (5), generally work for any return sample, and establish approximate relationships between the geometric and arithmetic averages and the variance. These formulas are all based on Taylor series expansions up to the second degree.

The question as to "Which is best?" is a debate for another paper as all three of these approximations provide values for *G* which begin decreasing at some level of *V* in the domain of values for *V*. All three approximations see the curve for *G* bend downwards at some point as *V* increases.

For various, real-world data sets of various distributions, we find that the value for Equation (3), though for some data sets not the best estimation for *G* of the three, is *never* the worst estimation for *G* of the three equations. For this reason alone, in the real-world where often, the distribution of *a priori* data is unknown, Equation (3) is in fact, the best estimator for *G*.

Finally, when we get into the lower values for the cosine of *SD* over *A*, that is, when *V* begins to get larger with respect to *A*, when the points of geometrical significance are encountered, particularly by the time *V* is at a critical enough value to encounter the point ψ, Equations (4) and (5) are unable to keep pace. For the 2: -1-coin toss, this becomes evident in **Figure 6 and 7**.

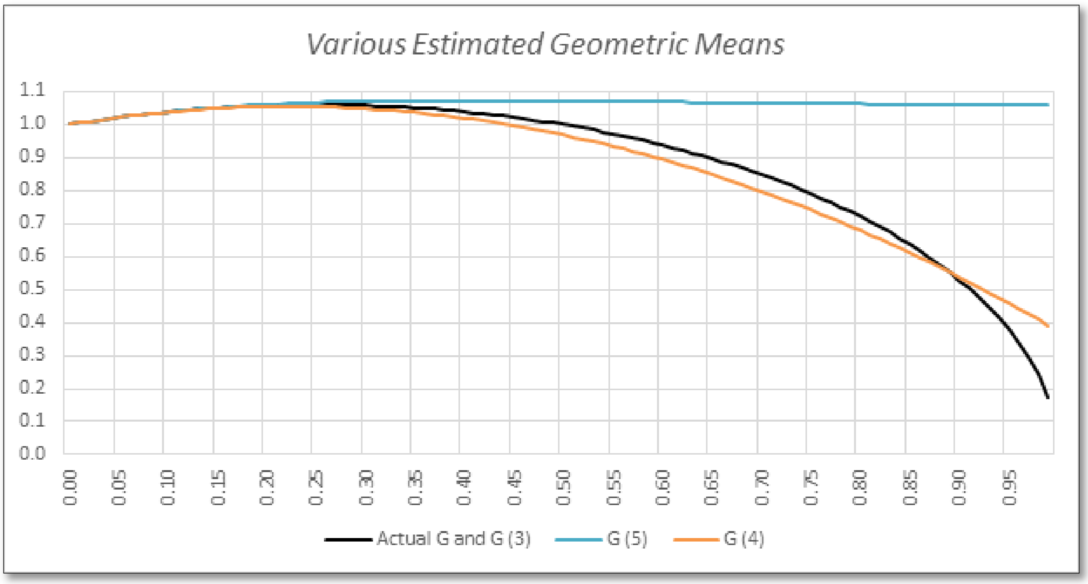

Figure 6. Three variants for estimating G in the 2: -1-coin toss, all of which bend downwards at some increased V. The Vertical scale here is G= 0 to 1.1.

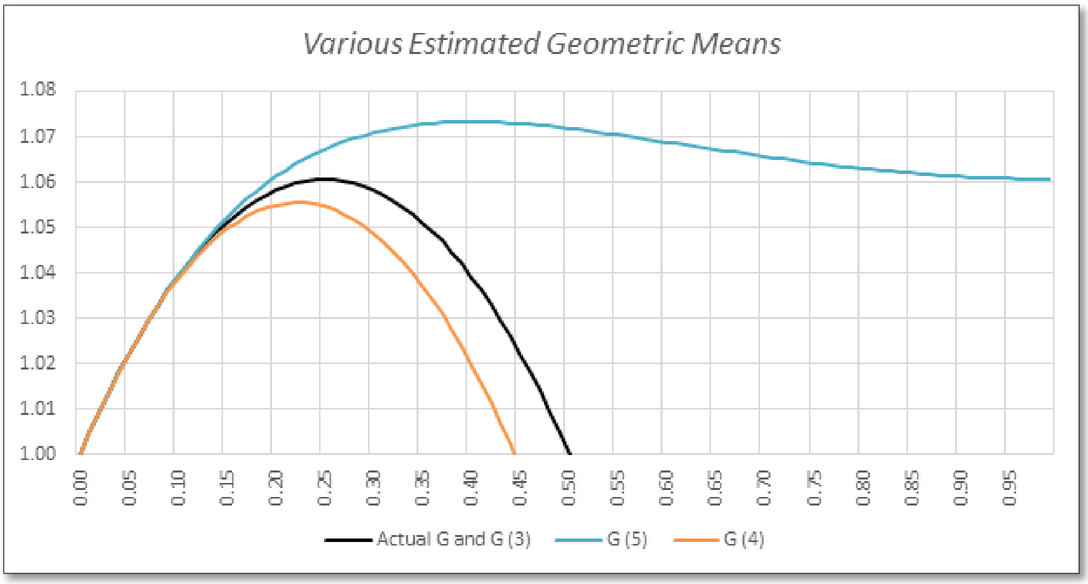

Figure 7. Three variants for estimating G in the 2: -1-coin toss, all of which bend downwards at some increased V. This is the same as Figure 6, but here the vertical scale is G= 1.0 to 1.08.

This is central to our argument, that at some point of higher V, G is decreased. All three values for estimating G support this contention. However, we find that using the Pythagorean estimate of Equation (3) to be the best fit for our purposes and can be certain, regardless of the distribution of outcomes we will be applying it to, it will never be the worst estimate, and most importantly, that by increasing variance, at some increased value (perhaps immediately) we will begin to see reductions in G.

Examples

Example 1 – A politically agnostic means of deficit reduction

For the sake of illustration, we demonstrate the method in a simplified example using a binary stream of stochastic outcomes. As pointed out, this stream need not necessarily be binary-distributed, we merely use it for simplicity's sake to illustrate the method.

We examine the period-on-period percentage changes in growth and re-express these as a *Multiple* on the aggregate of the prior period as 1 plus the period's percentage growth.

We further introduce in this example the notion of *Constant Dollars Forward*, which represents the amount of cumulative debt (it's initial principal plus its service) represented by an individual *silo* of one million going forward, as of a given start date. Thus, each period forward will see new, non-debt-service borrowing that is not addressed here. Rather, this addresses the debt-service cost of one million in debt into the future.

Our first example sequence shows a constant increase of 99.5% per period in debt service, which appears as follows for ten periods **(Table 1)**.

Table 1. Constant increase of 99.5% per period in debt service

Period	Percent	Multiple	Constant Dollars Forward
0			\$1,000,000
1	99.5%	1.995	\$1,995,000
2	99.5%	1.995	\$3,980,025
3	99.5%	1.995	\$7,940,150
4	99.5%	1.995	\$15,840,599
5	99.5%	1.995	\$31,601,995
6	99.5%	1.995	\$63,045,980
7	99.5%	1.995	\$125,776,730
8	99.5%	1.995	\$250,924,577
9	99.5%	1.995	\$500,594,531
10	99.5%	1.995	\$998,686,088

Since the stream is the same 99.5% from one period to the next, the arithmetic average, *A*, and the geometric average, *G*, of the period-on period multiples is identical at 1.995. Thus, for this constant growth sequence, we have:

A=1.995;
V=0
SD=0
G=1.995

There is no variance in this first sequence. In this example, we are able to keep *A* constant while increasing variance, *V*, and so the introduction of any variance – that is, increasing the base of a right-angled triangle while holding the length of the hypotenuse constant, must result in a shorter vertical leg, *G*.

We see this when we amend the sequence to be simply the alternating binary distribution of 100% and 99% increases **(Table 2)**.

The introduction of variance, even as small as .000025 in the period-on-period growth multiples, shows a growth rate diminished from 1.995 when variance, *V*, was 0 to 1.994993734 even though the (arithmetic) average period-on-period growth multiple remains at 1.995, or an average gain of 99.5% per period. The cumulative growth at the end of the period is reduced simply by the introduction of variance.

Table 2. Amended sequence.

Period	Percent	Multiple	Constant Dollars Forward
0			$1,000,000
1	100.0%	2	$2,000,000
2	99.0%	1.99	$3,980,000
3	100.0%	2	$7,960,000
4	99.0%	1.99	$15,840,400
5	100.0%	2	$31,680,800
6	99.0%	1.99	$63,044,792
7	100.0%	2	$126,089,584
8	99.0%	1.99	$250,918,272
9	100.0%	2	$501,836,544
10	99.0%	1.99	$998,654,723
	A=	1.995	
	V=	0.000025	
	SD=	0.005	
	G=	1.994993734	

Unlike **Figures 1, 3, and 4**, where the arithmetic average growth rate, A, increased with respect to variance, V, and hence the geometric average growth rate, G, had a peak farther down range than the leftmost values on the horizontal axis of $f=0$ or $cos(SD/A)=1$, when A does not increase with increased V (as may be the case in real-world applications of directed efforts to increase period-on-period variance) we find G achieves its peak at the leftmost values and decreases from there with increased directed effort.

To continue to increase this period-on-period variance we find, still holding the (arithmetic) average growth per period at 99.5%, represented as the binary sequence 200, -1 **(Table 3)**.

The period-on-period growth rate, G, is further reduced to 1.723368794, and the total growth of the starting amount of $1,000,000 is now cut to less than $1/4^{th}$ over ten periods of what it was at no period-on-period variance.

Assuming we can increase variance without bound, we examine the sequence 272, -73, which shows a period-on-period average growth rate, A, of 1.995 and variance, V, increased to 2.975625 **(Table 4)**.

Notice the Constant Dollars Forward has barely increased over this sequence of 10 periods. We increase the variance by one more degree at the sequence 273, -74, which still has an arithmetic average growth rate, A, of 1.995 (Table 5).

We now see our geometric average growth rate, G, to be less than 1 at 0.98478424 for this sequence. We are now past the point ψ in terms of the size of the base, SD, with respect to the hypotenuse, A, and the sequence has now thus become terminal. This is so even though this sequence with an A of 1.995 thus grows at an (arithmetic) average growth of 99.5% per period. It is still, by virtue of increased variance, terminal.

Table 3. Amendments to continue increase of period-on-period variance.

Period	Percent	Multiple	Constant Dollars Forward
0			$1,000,000
1	200.0%	3	$3,000,000
2	-1.0%	0.99	$2,970,000
3	200.0%	3	$8,910,000
4	-1.0%	0.99	$8,820,900
5	200.0%	3	$26,462,700
6	-1.0%	0.99	$26,198,073
7	200.0%	3	$78,594,219
8	-1.0%	0.99	$77,808,277
9	200.0%	3	$233,424,830
10	-1.0%	0.99	$231,090,582
	A=	1.995	
	V=	1.010025	
	SD=	1.005	
	G=	1.723368794	

Table 4. Increasing variance without bound and variance increased.

Period	Percent	Multiple	Constant Dollars Forward
0			$1,000,000
1	272.0%	3.72	$3,720,000
2	-73.0%	0.27	$1,004,400
3	272.0%	3.72	$3,736,368
4	-73.0%	0.27	$1,008,819
5	272.0%	3.72	$3,752,808
6	-73.0%	0.27	$1,013,258
7	272.0%	3.72	$3,769,320
8	-73.0%	0.27	$1,017,717
9	272.0%	3.72	$3,785,905
10	-73.0%	0.27	$1,022,194
	A=	1.995	
	V=	2.975625	
	SD=	1.725	
	G=	1.002197585	

Table 5. Variance increased by one more degree.

Period	Percent	Multiple	Constant Dollars Forward
0			$1,000,000
1	273.0%	3.73	$3,730,000
2	-74.0%	0.26	$969,800
3	273.0%	3.73	$3,617,354
4	-74.0%	0.26	$940,512
5	273.0%	3.73	$3,508,110
6	-74.0%	0.26	$912,109
7	273.0%	3.73	$3,402,165
8	-74.0%	0.26	$884,563
9	273.0%	3.73	$3,299,420
10	-74.0%	0.26	$857,849
	A=	1.995	
	V=	3.010225	
	SD=	1.735	
	G=	0.98478424	

Thus, an initial silo of the growth of $1,000,000 in debt service has been broken, and at the current average growth rate and variance will diminish to nothing in time. Even if the average growth rate, *A*, increased with respect to time, there still exists a value for *V* which will cause the process to self-cannibalize.

Variance in a nation's cumulative debt is affected by tax policy, sporadic principal paydowns, and rates of borrowing which are, at the shorter maturities at least, influenced by the policies of central banks.

The technique is politically agnostic, and there is no reason why the period-on-period variance should not be among the primary considerations of policymakers in such matters. These are economic gains to the population at large that are heretofore being foregone, and would be acceptable to all regardless of politics. Further, the matter is rife in potential fruit for medical and pharmacological researchers.

However, lawmakers and central bank policy makers - practitioners working to diminish a nation's cumulative debt, may rightfully object that such an example disregards a plethora of real-world constraints they are up against. Similarly, applications in the biological sciences – epidemiology, pharmacological dosing, etc., may make similar arguments against the technique (e.g., the human body could not survive the higher doses of a drug necessary in certain periods required to increase period-on-period variance).

These are all valid arguments. The purpose of this paper is to introduce the technique in pure, theoretical form, that actual application left to the practitioners where the technique *might* be applied. Nevertheless, with these caveats in mind we examine a naïve, hypothetical epidemiological example of how this principle can be useful.

Example 2 – Applications involving malevolent cellular growth

In the following example, we consider a bacterial culture in a petri dish which we wish to diminish via an antibiotic. Let us assume the growth rate of this culture is 1.08 per unit time. That is, it is increasing in size by a multiple of 1.08 as each unit of time transpires. In other words, it grows 9% in size for every unit of time.

If left untreated after ten units time, we find it has therefore grown to:

$G = 1.08^{10}$=2.158925 times its original size

Further, since it is left untreated, we determine its variance, *V*, in period-on-period growth to be *0*, hence, the arithmetic average growth rate, *A*, is equal to *1.08* and the geometric average growth rate, *G*, is per Equation (3).

Generally, antibiotics are prescribed in a manner so as to induce a constant blood level of the antibiotic. The effect, in terms of this perspective is that we have reduced G while leaving V still 0, and hence have obtained this resultant lower geometric average growth rate (the only factor in this perspective we observe directly in this example), *G*, solely by reducing *A*, the arithmetic average growth rate.

Mathematically, we know two things:

1. Any increase in standard deviation, *SD*, in period-on-period growth (which is the square root of *V*) is akin to a *decrease* in *A*, the arithmetic average period-on-period growth rate. Thus, if we can increase *V* at a faster rate than *A* increases, we gain by this action in terms of diminishing the average growth rate per period, *G*.

   However, since usually in increasing variance, the average arithmetic growth rate, A, also increases

2. At that point where $A^2 - V < 1$, we have reached point ψ where growth diminishment is assured by virtue of this increase variance as shown in **Figure 8**.

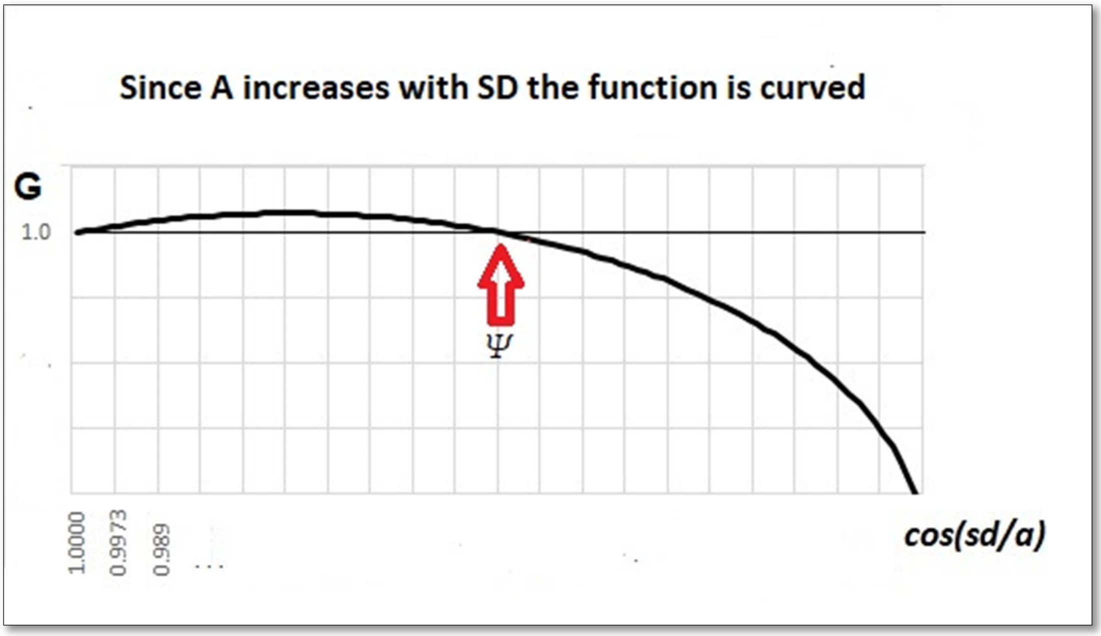

Figure 8. Since A tends to increase as SD increases when we alter a blood level to a non-constant state, the function curves over and at a certain critical point, ψ, where the values are increased. At ψ the function for the growth rate, G, crosses below 1 and hence diminishment is assured.

To invoke variance, we must now administer the antibiotic in a manner which is counter-intuitive and administer it in a manner where we do not induce a constant blood level.

We shall assume in this hypothetical example that the prescribed dosage for this type of bacterial culture at this level would be to apply 100 mcg of the particular antibiotic per unit time.

In maintaining a "constant blood-level" of antibiotic, by applying the prescribed amount per unit time consistently, we have minimized variance. This is the common manner in which antibiotics are used.

As time elapses, we can measure the amount of culture in the petri dish to determine the growth rate per unit time. Let us suppose we find that after 10 units of time, our culture has shrunk to 60% of its original size from when we began applying 100 mcg of antibiotic per unit time.

Thus, the growth rate, *G*, can be determined to be:

$G = 0.6^{(1/10)} = 0.6^{.1} = .9502$

Thus, applying 100 mcg per unit time sees the culture grow as the size of the culture for the previous time unit times 0.9502.

Looking at this then, since our Variance, *V*, is 0, then our arithmetic growth rate per unit time, determined from Equation (3) is:

$$A^2 = G^2 + SD^2 = G^2 + V$$

Since V=0, then A=G in this case, and thus A=.9502 also.

We see then that without 100mcg per unit time applied to the culture, the culture will grow at a rate of 1.08 per unit time, and with the 100mcg constant-level antibiotic treatment, it will grow at a rate of .9502 per unit of time (*i.e.*, since it grows by its multiple per unit time, it "shrinks" per unit time when the multiple is less than 1).

Given the nature of this example, we do not see the arithmetic average growth rate directly but rather the geometric average growth rate. Hence, we must calculate the arithmetic average growth rate from the geometric average and the variance in period-on-period growth.

Furthermore, since we also cannot examine the variance directly either (because the variance we witness is in period-on-period geometric growth, not the arithmetic data underlying it from which the variance must be determined) we are, in effect, blind to the inputs of A and V (or SD) in the critical Equation 3 from which we determine that A^2-V will give us that growth-diminishment critical point of ψ where growth-diminishment is mathematically assured.

Nevertheless, we can extrapolate, for dosages and schedules of sporadic administration what the curve for G, the period-on-period growth rate as depicted in **Figure 8** may be.

We induce variance through such a seemingly heterodox schedule of administration. Any heterodox schedule can be attempted, plotted and the critical points discerned. In addition to the point ψ, since increasing variance initially increases *A* at a rate such that *G* increases (as depicted in **Figure 8**), the peak of the curve for G with respect to the cosine(SD/A) is too, a geometrically-critical point representing that point where increases in variance, *V*, now begin to decrease the growth rate, *G*.

We demonstrate here by a very simple, hypothetical schedule with naïve assumptions (so as to sketch out the principle). Rather than administer our recommended dosage of 100mcg for every period, we shall administer twice that (with the naïve assumption that it will give us twice the shrinkage in growth as with 100 mcg), and administering this only every other period (and thus the assumption that the growth for these intervening periods is the growth rate, if left entirely untreated, of 1.08).

We sketch this hypothetical for illustrative purposes **(Table 6)**.

Table 6. A hypothetical schedule for illustrative purposes.

Period	Percent	Period Multiple	Culture Size Multiple
0			1
1	-9.96%	0.9004	0.9004
2	8.00%	1.08	0.972432
3	-9.96%	0.9004	0.875577773
4	8.00%	1.08	0.945623995
5	-9.96%	0.9004	0.851439845
6	8.00%	1.08	0.919555032
7	-9.96%	0.9004	0.827967351
8	8.00%	1.08	0.894204739
9	-9.96%	0.9004	0.805141947
10	8.00%	1.08	0.869553303
	A=	0.9902	
	V=	0.00806404	
	SD=	0.0898	
	G=	0.986119668	

And after 10 periods of such alternating double-treatments, we find growth has been reduced by nearly 13%, depicted in **Figure 9**.

This simple, hypothetical schedule is but one schedule – an infinite number are available and if the host can sustain the schedule, then there exists a schedule of administration more effective than the accepted, outright, constant-blood level recommend dosages. Mathematically, therefore, there exists many schedules that are more beneficial provided the host can sustain the schedule.

We have been discussing the growth of bacteria in a petri dish, but the same notion can be extended to a bacterial infection in a human being, e.g., an upper-respiratory infection and antibiotic treatment. Again, the critical caveat is if the host can sustain the schedule being the limiting factor. Different antibiotic treatments can be handled in varying doses differently, and thus the principle of increasing the variance also affects which antibiotic to invoke.

Further, not only is the principle applied with this caveat to bacterial infections but other malevolent cell function conscriptions as well, such as cancer and potential chemotherapy dosages and abilities to sustain higher dosages, with some chemotherapies and doublets thereof more easily sustained at the varying dosages than others, thus becoming of possibly greater concern, ceteris paribus.

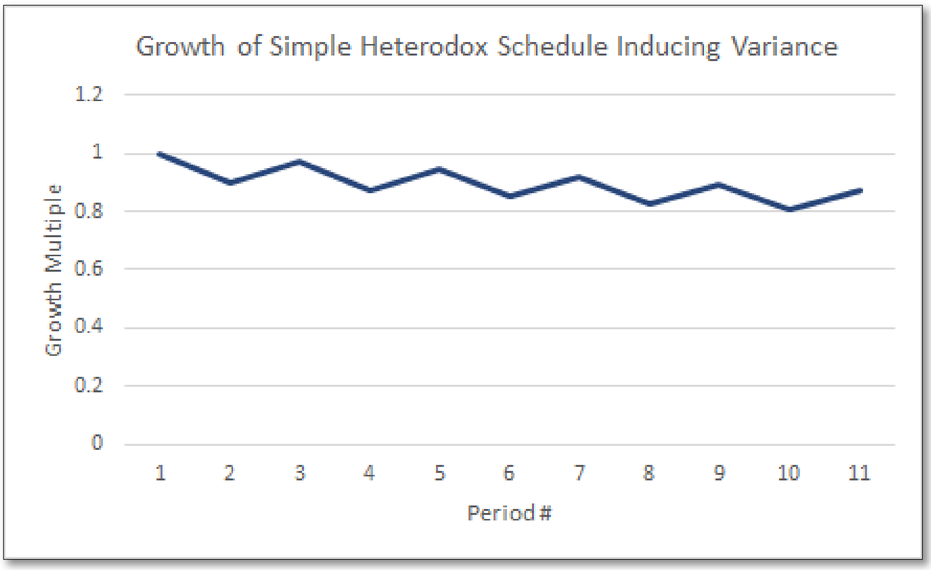

Figure 9. Growth of applying 200 mcg antibiotic every other period to induce variance beyond ψ. The growth rate assures total eventual diminishment.

Example 3 – Pathogenic spread through a population

Thus far we have looked at an example of an out of control national debt, as well as certain infections and malevolent cellular conscriptions within an organism. We can also apply these simple properties to the growth function innate in a pathogenic outbreak between organisms themselves. This too is a geometric growth function, and the effect of increasing the period-on-period variance in growth to tame the function applies as well.

Let us consider a population of organisms, human beings in the immediate case, where a viral outbreak occurs. The problem in application of these properties in all cases – not just the examples cited here in is two-fold:

1. How can we induce greater period-on-period variance (*i.e.,* what actions can we take to migrate rightward on the horizontal axis of Figures1,3,4,5,6,7, and 8)?

-and-

2. Can we induce variance safely enough that we can achieve the growth-terminal critical point ψ?

Inducing period-on-period variance can be achieved through various possible measures with the one(s) best suited determined by qualified epidemiologists. These, below, are a suggested sampling of possibilities:

1. *Quarantine*, much like an antibiotic or chemotherapy in the malevolent cellular growth example, a sporadic period between serious, hard lock-down and none-at-all.

2. *Prophylactic measures (usually pharmacological)* even if only to segments of the population sporadically, to induce variance.

3. *Vaccination* - which, when applied to the entire population, liberates the population from that pathogenic event, but even if to only part of the population will induce variance.

These measures can provide a viable stop-gap for situations involving limited resources if they induce enough variance in the period-on-period growth rate of infected individuals within a population.

If we can induce variance in the spread of a pathogen within a population and do so safely such that the invoked variance is of *degree sufficient to achieve the growth-terminal critical point ψ*, the growth curve is broken.

In the particular case of attempting to invoke variance in period-on-period growth to mitigate the spread of a pathogenic outbreak, it is vital to avoid entropic outcomes. That is to say, if one measure is being used to induce variance, other measures must not interfere in such. In inducing variance, a wave function is created on what otherwise is a simple curved function of increasing spread. If multiple wave functions are overlaid such that peaks and troughs cancel, which, conceivably could occur if multiple avenues of inducing variance are invoked, especially in uncoordinated fashion, the campaign to reduce the spread, to break the growth function through increased period-on-period variance in growth, will fail.

CONCLUSION

We have demonstrated how increasing variance in geometric growth rates can diminish geometric growth. We find here the aspects in the underlying data that are related to the geometric growth rate by a Pythagorean or closely-Pythagorean relationship, depending on the distribution of outcomes. These three components are the arithmetic mean growth rate, A, the geometric mean growth rate, and the variance (or standard deviation in growth rates). When the arithmetic growth rate squared less the resultant variance is 1 or less, the growth will ultimately cease (as multiplying any product repeatedly by a number less than 1 yields a new product approaching zero).

We examined growth rates with respect to a fraction of resources to allocate to risk, or in proxy of that, growth rates with respect to the cosine of the standard deviation in period-on-period outcomes and their arithmetic average. Often we only have the geometric average period-on-period growth rate, and must sketch out what the variance or standard deviation is, as well as the arithmetic average growth rate, and where this is not feasible, we must seek a schedule of increasing variance until the growth-terminal critical point ψ is arrived at.

We have looked at examples which are either politically agnostic, or uses less resources, or heterodox approaches than conventionally used for "bending the curve" of malevolent geometric growth. We have examined simple, hypothetical examples where this might be applied in terms of a nation's aggregate debt, in terms of malevolent cellular growth, and in terms pathological spread through a population. Each example also is riddled with specific concerns, such as "can the host survive increased, sporadic doses," as well as avoiding entropic outcomes in more social applications such as pathological spread in a population.

The examples provided, though hypothetical and naïve to their field of application, are provided as trail heads, as a means of sketching out how one might go about attempting to find a means to apply the principle of increased variance and to highlight potential pitfalls to be avoided for the particular application.

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